

A complete worksheet is worth extra points in your quiz grade. Due on Friday 30 January 2009. Show your work and be as *clear as possible*. Don't just treat the blank space as a scattered jumble of math; try to do your work linearly, with “=” symbol between steps so that I can follow what you're doing.

By the end of this worksheet, we will be able to simplify things like

$$\frac{x^{-4}y^2x^3(x^2y^3)^5}{y^2x^{-1}x(xy)^{-1}y^2}.$$

This is a big problem, to be sure. But if we break it up into smaller steps, we can make even the large and annoying into something manageable and easy. In this worksheet, we're going to build our way up to simplifying things like this.

The basic idea

When it comes to simplifying, the main idea is that $x^ax^b = x^{a+b}$. This idea guides everything else we will be doing, so it's best that we get quick with it!

Here are some examples of this rule in action:

- (a) $x^4x^5 = x^9$
- (b) $xx = x^2$
- (c) $x^3x^{-1} = x^2$
- (d) $x^{-2}x^{-3} = x^{-5}$

In each case, we just looked at the exponents and added them up. Nothing fancy: just add them.

The important thing is that this only works when the base is the same. For instance, we can *not* simplify x^3y^2 into anything else, since the x and the y are *not the same variable*.

1. (3 points) Try this out for yourself. Simplify each of the following:

(a) x^3x^3

(b) y^7y^2

(c) x^2x^{-4}

What if there are many variables involved? The same rule applies, but we need to do some preparation before we can use it.

If you have something like $x^2y^4x^3y^2$, it's best to move the x 's and y 's around so that they are next to each other, and then apply the simplification rules. For instance,

$$\begin{aligned}x^2y^4x^3y^2 &= x^2x^3y^4y^2 && \text{Rearrange the } x\text{'s and } y\text{'s.} \\ &= x^5y^6 && \text{Simplify using the tricks you know.}\end{aligned}$$

2. (3 points) Try this out for yourself. Simplify each of the following:

(a) $x^4y^{-2}x^3y^2y^2$

(b) $y^1x^1y^1y^2x^4x^{-3}$

(c) $y^2x^3z^4x^2zy^9$

Fractions show up

Sometimes you need to simplify an expression that has fractions in it. There are tricks for manipulating such things, and if you get good at them you will find that fractions don't make the problem any harder.

The basic rule here is that

$$x^a = \frac{1}{x^{-a}}, \text{ or alternatively } x^{-a} = \frac{1}{x^a}.$$

For instance,

(a) $x^{-2} = \frac{1}{x^2}$

(b) $x^3y^{-2} = \frac{x^3}{y^2}$

(c) $\frac{1}{y^{-4}} = y^4$

(d) $\frac{1}{z^3} = z^{-3}$.

If you have a problem that involves fractions, you can always use these rules to get rid of the fraction! Here is an example of what this looks like:

$$\frac{x^2y^4z^3}{x^2y^{-2}z} = (x^2y^4z^3)(x^{-2}y^2z^{-1}).$$

If we had to simplify, we could now just group the variables together and add their exponents, just like we did on the previous section.

3. (3 points) Try this out for yourself. For each of these problems, (1) get rid of the fraction, (2) simplify, (3) write the answer as a fraction.

(a) $\frac{x^3}{x^2}$

(b) $\frac{y^7}{y^{-3}}$

(c) $\frac{xy^2}{x^{-2}y^2}$

Working with parenthesis

It's extremely common to have to simplify something that looks like this:

$$x(yx)^2.$$

It's very important in these problems to be able to eliminate the parenthesis. The rule goes like this:

$$(xy)^a = x^a y^a.$$

This is really just a consequence of some of the rules we already for. For instance,

$$(xyz)^3 = (xyz)(xyz)(xyz) = xxx yyy zzz = x^3 y^3 z^3.$$

It's also important to know what happens when there are powers *inside* the parenthesis. As a basic rule, they multiply:

$$(x^2 y^3)^2 = (x^2)^2 (y^3)^2 = x^4 y^6.$$

4. (*4 points*) Try this out for yourself. Simplify each of the following. Write the answer as a fraction if possible:

(a) $(x)^3$

(b) $(xy)^3$

(c) $(x^6y^2)^2$

(d) $(x^{-2}y^4)^{-1}$

Building up

We're now ready to simplify some bigger problems.

5. (*12 points*) Try this out for yourself. Simplify by performing the following steps: (1) If it's written as a fraction, get rid of the fraction. (2) Simplify by grouping variables and adding powers. (3) Write it out as a fraction again if possible. Your answer should have each variable appearing only once.

(a) $\frac{x^{-2}x^4y^4}{y^{-2}}$

(b) $\frac{yxy}{(yx^2)^2}$

(c) $\frac{y^{-2}x^{-3}z^{-3}}{w^{-9}}$

(d) $\frac{x(yxx^2)^3}{(y^3x^{-1})^{-2}}$

The Master Class

6. (25 points) Now simplify $\frac{x^{-4}y^2x^3(x^2y^3)^5}{y^2x^{-1}x(xy)^{-1}y^2}$ using whatever method you like (although the method we used in the last problem isn't a bad start!)