

Math 1030 Section 7

Final Review

Fall 2008

1 Unit conversion

Problem 1. *You need to water a lawn that measures 212 feet by 49 feet. The law requires 0.29 gallons of water per square yard each time you water. How much will you pay each time you water the lawn if a City Corporation charges \$0.007 per gallon?*

$$0.007 \frac{\text{dollars}}{\text{gallon}} \cdot 0.29 \frac{\text{gallons}}{\text{yard}^2} \cdot (212 \cdot 49 \text{ft}^2) \cdot \frac{1 \text{ yard}^2}{3 \cdot 3 \text{ft}^2} = 2.34 \text{ dollars.}$$

Problem 2. *You plan to travel to Europe. You will take a 1,553 kilometer trip and you find out that the cost of gas is \$4.89 per gallon. The car that you would like to rent uses 1 liter of gas for every 19 kilometers that you drive. How much money will you spend on gas on this trip? (Remember that 1 liter is 0.2642 gallons).*

$$4.89 \frac{\text{dollars}}{\text{gallon}} \cdot \frac{0.2642 \text{ gallons}}{1 \text{ liter}} \cdot \frac{1 \text{ liter}}{19 \text{ km}} \cdot 1553 \text{ km} = 105.60 \text{ dollars.}$$

2 Interest

Problem 3. *If you deposit \$459 now and you can get an APR of 3.59% compounded quarterly, how much will you have in 16 years?*

$$459 \cdot \left(1 + \frac{0.0359}{4}\right)^{4 \cdot 16} = 813.13.$$

Problem 4. *If you deposit \$150 now and you can get an APR of 3.41% compounded continuously, how much will you have in 15 years?*

$$150 \cdot e^{0.0341 \cdot 15} = 250.17.$$

3 Exponential change

Problem 5. *Your car is worth \$31,205. If the value of your car is depreciating at a rate of 18% per year, how much will your car be worth 8 years later?*

$$31,205 \cdot (1 - 0.18)^8 = 6,378.74.$$

Problem 6. *This is about a computer virus. On 1 January 2008 the virus had already infected 4,592 computers. The number of infected computers increases by 82% per day. How many computers will be infected on 23 January 2008?*

$$4592 \cdot (1 + 0.82)^{23} = 4.4019368 \cdot 10^9.$$

4 Percentages

Problem 7. *The smoking rate among a certain population jumped 63%, up to 13%. What percentage of the population smoked before the increase? Round to the nearest integer percentage.*

Let's say that x portion of the population smoked before. Evidently we have the equation

$$x \cdot (1 + 0.63) = 0.13$$

so that, solving for x we get $x = 0.08$. Then apparently 8% of the population smoked before the increase.

Problem 8. *A 2008 Porsche 911 can be purchased for \$81,478. A 1998 Lamborghini Diablo costs \$245,623. What percentage more does the Lamborghini cost than the Porsche?*

$$\frac{245,623 - 81,478}{81,478} = 0.668$$

so that the Lamborghini costs 66.8% more than the Porsche.

5 Geometry

Problem 9. *I have a model car. Every dimension of the model is $1/18$ of the dimension of the car it models. So we say that it is a $1/18$ scale model. Curious about the volume of the car, I set up an experiment: I bought a tank that measures 4 feet by 3 feet and filled it with water. When I put the model into the tank, the water level rose by 0.24 inches. What is the volume of the real car? Give your answer in cubic feet.*

We begin by computing the volume of the toy car. We let x be the water level in the tank before the toy was submerged. Then the volume of the car is given by

$$\begin{aligned} 4\text{ft} \cdot 3\text{ft} \cdot (x\text{ft} + 0.24\text{in}/(12\text{in}/\text{ft})) - 4\text{ft} \cdot 3\text{ft} \cdot x\text{ft} &= (4 \cdot 3 \cdot 0.24/12)\text{ft}^3 \\ &= 0.24\text{ft}^3. \end{aligned}$$

Since the toy is a $1/18$ -scale model, the volume of the real car is 18^3 times that of the toy, which is $0.24\text{ft}^3 \cdot 18^3 = 1399\text{ft}^3$.

Problem 10. *The water drum of the water tower in Kuna, Idaho has the shape of a right circular cylinder with a semi-sphere top. The radius of the cylinder is 3.2 meters. The height of the cylinder (not including the semi-sphere cap) is 6.0 meters. Water flows into the tank at a rate of 0.051 cubic yards per second. How many minutes will it take until the tank is full? Give your answer in terms of minutes.*

We first find the volume of the water tower, which will tell us how much water is needed to fill the tower. The volume of the tower is just (volume of the cylinder) + (volume of the cap). So we get

$$\pi(3.2\text{m})^2 \cdot 6\text{m} + \frac{1}{2} \frac{4}{3} \pi(3.2\text{m})^3 = 261.65\text{m}^3.$$

Converting the above into cubic-yards, we get that the volume is 342.23yd^3 . Then it will take $\frac{342.23\text{yd}^3}{0.051\text{yd}^3/\text{s}} = 6,710.39\text{s}$ to fill the tank. This is 111.84 minutes.

6 Linear modeling

Problem 11. *Tomorrow morning there will be 3 inches of snow on the ground. Then the winter storm will start and snow will accumulate at a constant rate of 4 inches every 3 hours. How long will it take for the height of the snow to reach 14 inches?*

So after t hours, the number of inches of snow on the ground is given by $4(t/3) + 3$. Then to answer the question we just need to solve for t :

$$\begin{aligned}14 &= 4(t/3) + 3 \\11 &= 4(t/3) \\33 &= 4t \\ \frac{33}{4} &= t.\end{aligned}$$

Then it will take $33/4 = 8.25$ hours for the snow to reach 14 inches.

Problem 12. *In July 2000, you measured the circumference of a tree in the park. It was 15 inches. In July 2004, you measured that the circumference of the same tree was 31 inches. Assume that the circumference grew at a constant rate. How old will the tree be when its circumference is 141 inches?*

At first, you'd think we would need to know how old the tree was when the measurements were taken in July 2000. In the long run we won't need this information, but to setup the problem we will need to behave as if we *did* know the age of the tree in July 2000.

Ultimately we will develop an equation that tells us how big the tree will be when it is t years old. The problem tells us that the circumference grows at a constant rate, which we will call c . Our equation will be (circumference of tree) = $c \cdot t$, where t is measured in years.

Let's figure out what c is. Let T be the age of the tree in July 2000. Then in July 2004, the tree was $T + 4$ years old. The problem then gives us the following two equations:

$$\begin{aligned}15 &= cT, \\31 &= c(T + 4).\end{aligned}$$

Solving for c we get that $c = 4$ (we can get this by subtracting the first equation from the second). So now we will determine how old the tree will be when its circumference is 141 inches:

$$141 = 4t$$

so that $t = 141/4 = 35.25$ years.

7 Exponential modeling

Problem 13. *A community of rabbits has a doubling time of 3.4 months. How long will it take for the population to become 5.6 times the initial population? Give your answer in months.*

Since the doubling time for the population is 3.4 months, after t months the population will be $2^{t/3.4}$ times its original size. We want to know when the population will be 5.6 times its original size. So we seek to solve

$$5.6 = 2^{t/3.4}$$

for t . We could guess at values of t , or we can use logarithms:

$$\begin{aligned} 5.6 &= 2^{t/3.4} \\ \log 5.6 &= \log(2^{t/3.4}) \\ \log 5.6 &= (t/3.4) \log 2 \\ \frac{3.4 \cdot \log 5.6}{\log 2} &= t \\ 8.45 &= t. \end{aligned}$$

So the answer is 8.45 months.

Problem 14. *Suppose that 982 pounds of Pu-239 is deposited at a nuclear waste site in Southern Utah. The half-life of plutonium is 24,000 years. How much will be left after 110,574 years?*

We compute

$$982 \cdot \left(\frac{1}{2}\right)^{110574/24000} = 40.29$$

so that in 110,574 years there will be 40.29 pounds remaining.

8 Savings plan formula

Problem 15. *I would like to have \$1,940,994 dollars when I retire in 40 years. I have a savings account that pays an APR of 5.7% compounded 5 times per year. If I make regular deposits 5 times per year, how large should my deposits be?*

9 Finding half-life

Problem 16. *A population of rats decreases by 32.3% each month. Determine the half-life of the rat population.*

Problem 17. *A certain medication breaks down in the human body at a rate of 9% per hour. Determine the approximate half-life. Express your answer in terms of hours. Round your answer to the nearest tenth.*

10 Venn diagrams

Problem 18. *At an international conference on air quality, 43 of the 199 Americans were attorneys and 92 of the 129 non-Americans were attorneys. How many non-Americans were not attorneys?*

11 Loan payment formula

Problem 19. *You are buying a home for \$132,762. The bank offers you an APR of 6.3% on a fixed rate 30 year loan. The bank also offers you an APR of 5.3% on a fixed rate 20 year loan. How much less interest will you pay over the duration of the loan if you choose the 20 year loan instead of the 30 year loan?*